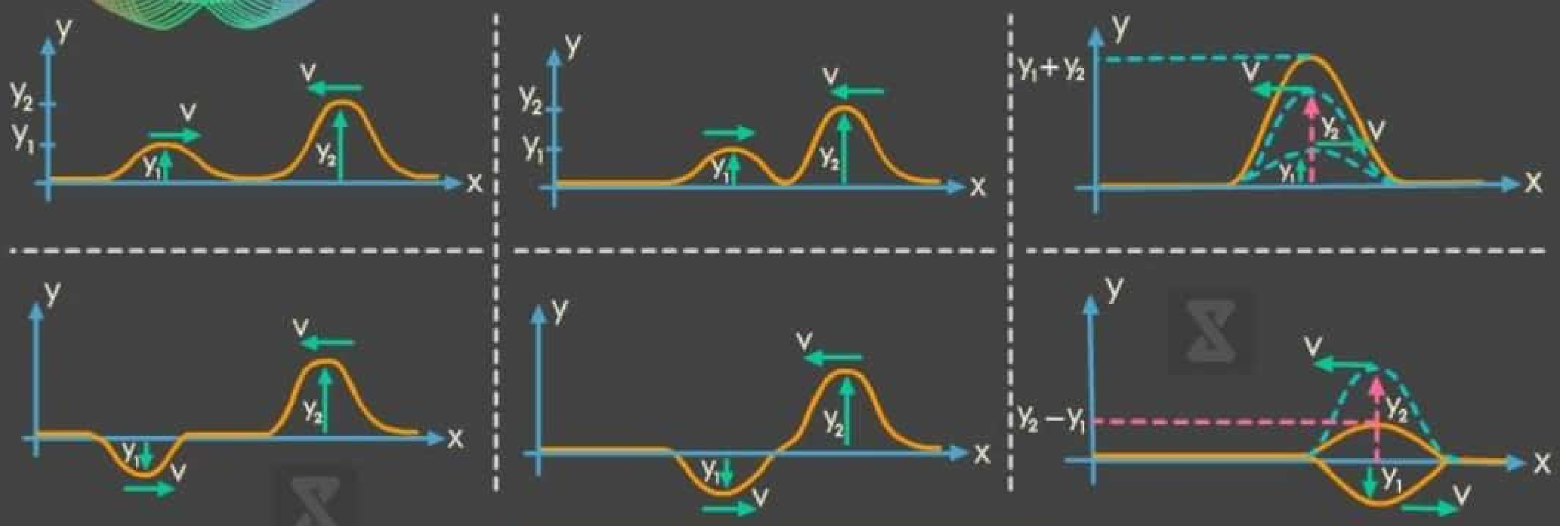
SUPERPOSITION AND STANDING WAVES

Part I

PRINCIPLE OF SUPERPOSITION

When two or more waves superpose on a medium particle then the resultant displacement of that medium particle is given by the vector sum of the individual displacements produced by the component waves at that medium particle independently.



INTERFERENCE OF WAVES

- If the two waves are exactly in same phase, that is the shape of one wave exactly fits on to the other wave then they combine to double the displacement of every medium particle **as shown in figure (a)**. This phenomenon is called as constructive interference.

- If the superposing waves are exactly out of phase or in opposite phase then they combine to cancel all the displacements at every medium particle and medium remains in the form of a straight line **as shown in figure (b).** This phenomenon is called as destructive interference.

Figure (a) ⟶ Constructive Interference

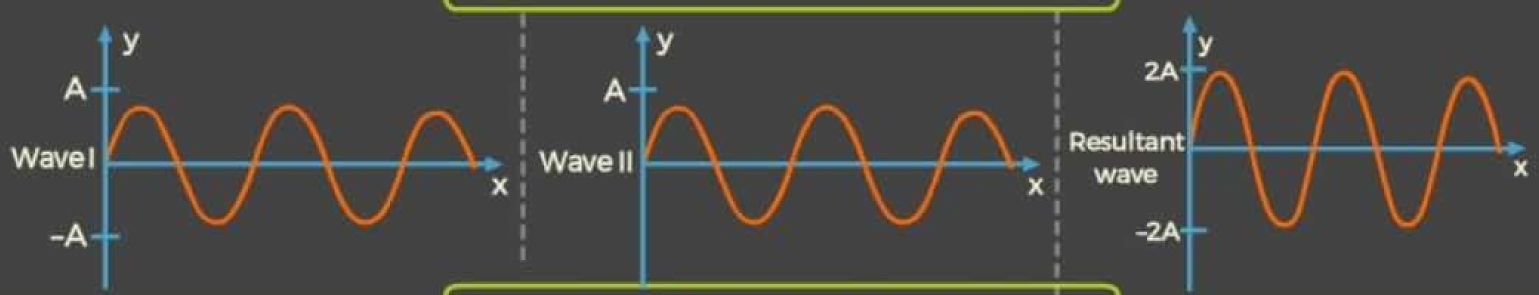
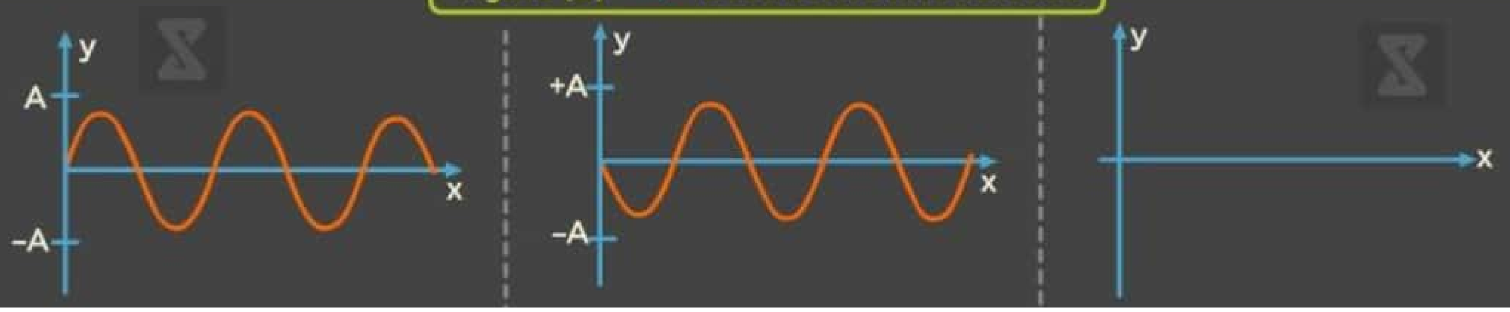


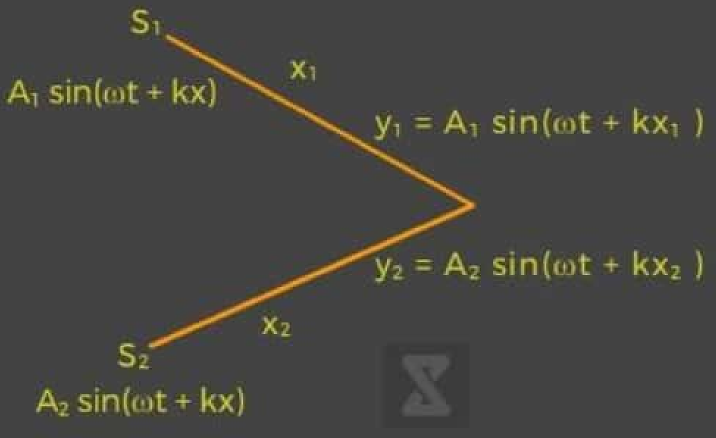
Figure (b) ⟶ Destructive Interference



SUPERPOSITION AND STANDING WAVES

Part II

ANALYTICAL TREATMENT OF INTERFERENCE OF WAVES



Whenever two or more than two waves superimpose each other, they give sum of their individual displacement.

$$y_1 = A_1 \sin(\omega t + kx_1)$$

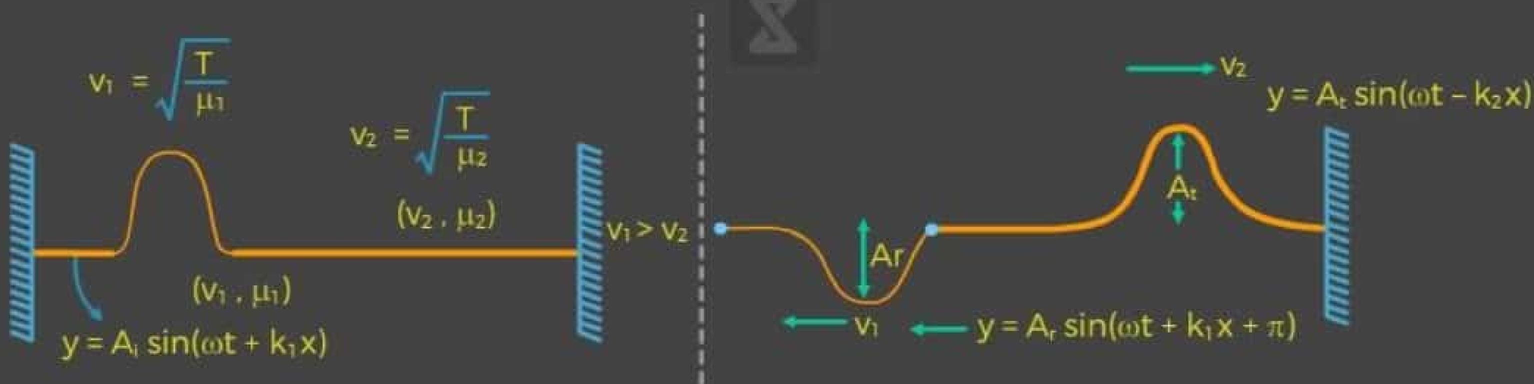
$$y_2 = A_2 \sin(\omega t + kx_2)$$

Due to superposition

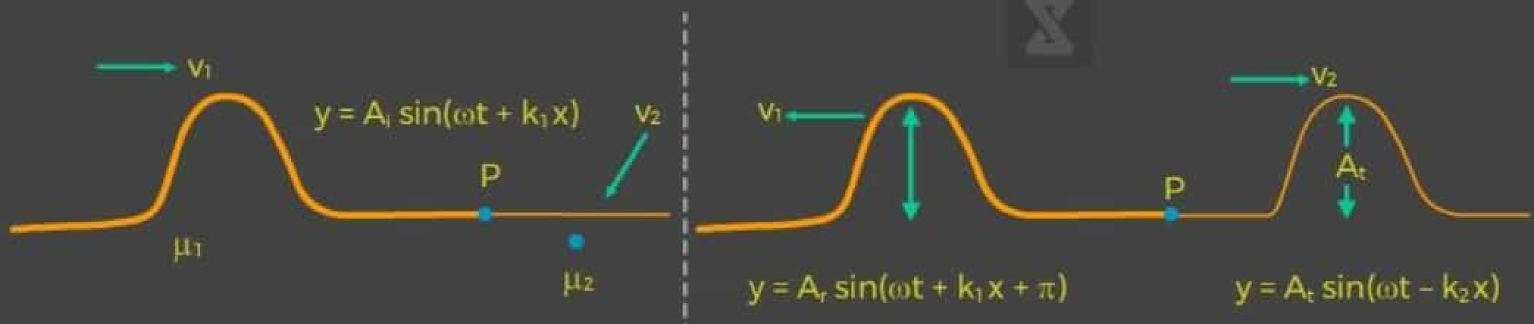
$$y_{net} = y_1 + y_2$$

REFLECTION AND TRANSMISSION BETWEEN TWO STRING

If a wave pulse is produced on a lighter string moving towards the friction, a part of the wave is reflected and a part is transmitted on the heavier string. The reflected wave is inverted with respect to the original one.



On the other hand if the wave is produced on the heavier string which moves toward the junction, a part will be reflected and a part transmitted, no inversion in waves shape will take place.



SUPERPOSITION AND STANDING WAVES

Part III

STANDING WAVES

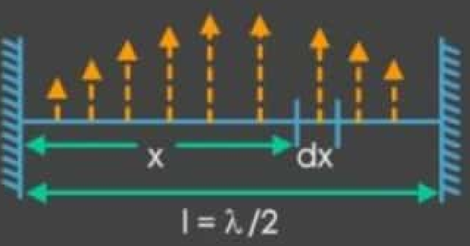
When two coherent waves travelling in opposite directions superpose then simultaneous interference of all the medium particles takes place. These waves interfere to produce a pattern of all the medium particles is what we call, a stationary wave.

ENERGY OF STANDING WAVE IN ONE LOOP

When all the particles of one loop are at extreme position then total energy in the loop is in the form of potential energy only. When the particles reaches its mean position then total potential energy converts into kinetic energy of the particles, so we can say that total energy of the loop remains constant.

Total kinetic energy at mean position is equal to total energy of the loop because potential energy at mean position is zero.

$$\text{Total K.E} = \frac{1}{2}\lambda A^2 \omega^2 \mu$$



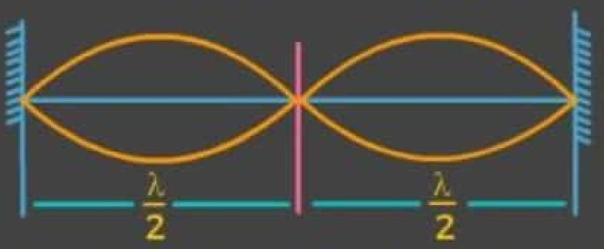
STATIONARY WAVES IN STRINGS

WHEN BOTH ENDS OF A STRING ARE FIXED

Fundamental Mode

The string vibrates in one loop in which the ends are the nodes and the centre is the antinode. This mode of vibration is known as the fundamental mode and frequency of vibration is known as the fundamental frequency or first harmonic.

$$f_1 = \frac{V}{2L}$$



First Overtone

The frequency f_2 is known as second harmonic or first overtone.

$$f_2 = \frac{V}{L}$$

Second Overtone

The frequency f_3 is known as third harmonic or second overtone.

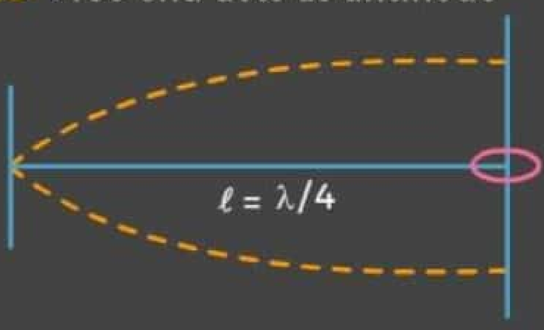
$$f_3 = \frac{3v}{2L}$$

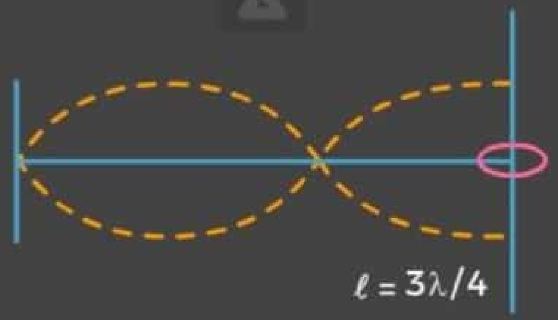
SUPERPOSITION AND STANDING WAVES

Part IV

- **When one end of the string is fixed and other is free**

Note- Free end acts as antinode





$f = \frac{1}{4\ell}\sqrt{\frac{T}{\mu}}$ fundamental or I^{st} harmonic

$f = \frac{3}{4\ell}\sqrt{\frac{T}{\mu}}$ III^{rd} harmonic or I^{st} overtone

In general : $f = \frac{(2n+1)}{4\ell}\sqrt{\frac{T}{\mu}}$ ($(2n+1)^{th}$ harmonic, n^{th} overtone)

S.No.	Travelling waves	Stationary waves
1.	These waves advance in a medium with a definite velocity	These waves remain stationary between two boundaries in the medium.
2.	In these waves, all particles of the medium oscillate with same frequency and amplitude.	In these waves, all particles except nodes oscillate with same frequency but different amplitudes. Amplitude is zero at nodes and maximum at antinodes.
3.	At any instant, phase of vibration varies continuosly from one particle to the other i.e. phase difference between two particles can have any value between 0 and 2π	At any instant, the phase of all particles between two successive nodes is the same, but phase of particles on one side of a node is opposite to the phase of particles on the other side of the node, i.e. phase difference between any two particles can be either 0 or π
4.	In these waves, at no instant all the particles of the medium pass through their mean positions simultaneously.	In these waves, all particles of the medium pass through their mean position simultaneously twice in each time period.
5.	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.